Ch-4 Square and Square Roots

Square of a number and square numbers

The square of a number is that number raised to the power 2.

Thus, if 'a' is a number, then the square of a is written as a^2 and is given by $a^2 = a \times a$.

That is, the square of a number is obtained by multiplying it by itself.

If a x a = b

i.e. $a^2 = b$, then we say that the square of number a is number b or the number b is the square of number a.

For example : 1) $3^2 = 3 \times 3 = 9$, so we say that the square of 3 is 9;

2) $(-4)^2 = -4 \times -4 = 16$, so we say that the square of -4 is 16;

3) $(3/5)^2 = (3/5) \times (3/5) = 9/25$ so we say that the square of (3/5) is 9/25;

Procedure to check whether a given natural number is a perfect square or not.

Step I: Obtain the natural number.

Step II: Write the number as a product of prime factors.

Step III: Group the factors in pairs in such a way that both the factors in each pair are equal.

Step IV: See whether some factor is left over or not. If no factor is left over in the grouping, then the given number is a perfect square. Otherwise, it is not a perfect-square.

Step V: To obtain the number whose square is the given number taken over one factor from each group and multiply them.

Examples on perfect-square

1) Is 225 a perfect-square? If so, find the number whose square is 225.

Solution : Resolving 225 into prime factors, we obtain

$225 = 3 \times 3 \times 5 \times 5$

Grouping the factors in pairs in such a way that both the factors in each pair are equal, we have

$225 = (3 \times 3) \times (5 \times 5)$

Clearly, 225 can be grouped into pairs of equal factors and no factor is left over.

Hence, 225 is a perfect-square.

Again, $225 = (3 \times 5) \times (3 \times 5)$

$= 15 \times 15 = 15^2$

So, 225 is the square of 15.

2) Is 150 a perfect-square? If so, find the number whose square is 150.

Solution: Resolving 150 into prime factors, we obtain

$150 = 2 \times 3 \times 5 \times 5$

Grouping the factors in pairs in such a way that both the factors in each pair are equal, we have

$225 = 2 \times 3 \times (5 \times 5)$

Clearly, 150 can be grouped into pairs of equal factors.2 and 3 factors are left over.

Hence, 150 is not a perfect-square.

Properties of square numbers

Property 1: A number having 2, 3, 7 or 8 at unit's place is never a perfect square. In other words, no square number ends in 2, 3, 7 or 8.

Example: None of the numbers 152, 7693, 14357, 88888, 798328 is a perfect square because the unit digit of each number ends with 2,3,7 or 8

Property 2: The number of zeros at the end of a perfect square is always even. In other words, a number ending in an odd number of zeros is never a perfect square.

Example: 2500 is a perfect square as number of zeros are 2(even) and 25000 is not a perfect square as the number of zeros are 3 (odd).

Property 3: Squares of even numbers are always even numbers and square of odd numbers are always odd.

Example: $12^2 = 12 \times 12 = 144$. (both are even numbers)

$19^2 = 19 \times 19 = 361$ (both are odd numbers)

Property 4: The Square of a natural number other than one is either a multiple of 3 or exceeds a multiple of 3 by 1.

In other words, a perfect square leaves remainder 0 or 1 on division by 3.

Example: 635,98,122 are not perfect squares as they leaves remainder 2 when divided by 3.

Property 5: The Square of a natural number other than one is either a multiple of 4 or exceeds a multiple of 4 by 1.

Example : 67,146,10003 are not perfect squares as they leave remainder 3,2,3 respectively when divided by 4.

Property 6: The unit's digit of the square of a natural number is the unit's digit of the square of the digit at unit's place of the given natural number.

Example : Example :

1) Unit digit of square of 146.

Solution : Unit digit of $6^2 = 36$ and the unit digit of 36 is 6,

so the unit digit of square of 146 is 6.

2) Unit digit of square of 321.

Solution : Unit digit of $1^2 = 1$, so the unit digit of square of 321 is 1.

Property 7: For every natural number n,

$(n + 1)^2 - n^2 = (n + 1) + n$.

Properties 8: The square of a number n is equal to the sum of first n odd natural numbers.

$1^2 = 1$	$2^2 = 1 + 3$
$3^2 = 1 + 3 + 5$	$4^2 = 1 + 3 + 5 + 7$ and so on.

Properties 9: For any natural number m greater than 1,

$(2m, m^2 - 1, m^2 + 1)$ is a Pythagorean triplet.

For any natural number m, we know that 2m, m2–1, m2+1 is a Pythagorean triplet.

$2m=16 \Rightarrow m=16/2=8$

$m^2-1=8^2-1=64-1=63$

$m^2+1=8^2+1=64+1=65$

$\therefore$ (16, 63, 65) is a Pythagorean triplet.